A Comprehensive Elastic –Plastic Micro Contact Behaviour of Rough Surfaces

A. Megalingam* and K.S.Hanumanthramji

Department of Mechanical Engineering
Bannari Amman Institute of Technology, Sathyamangalam,Erode - 638401, INDIA

Abstract

In precision applications, understanding the contact interactions among components are important to study the mechanical and tribological behaviors. Generally, contact interactions occur through contacting surfaces which are rough at microscopic level. Among various rough surface contact approaches, the single asperity based statistical approach is predominate one. In FEM based single asperity contact models like the Kogut-Etsion model (KE model), Jackson-Green model (JG model) and Shankar-Mayuram model (SM model), an axisymmetrical hemispherical asperity in contact with a rigid flat surface is modeled and analyzed using finite element concepts and the resultant contact parameters relations are extended to find rough surface contacts based on statistical approach but Megalingam-Mayuram model (MM model) developed a complete single asperity contact model but didn't extend to analysis the rough surface contacts due to complex empirical relations. In the present work, for the MM single asperity contact model, feasible dimensionless empirical relations are developed to calculate dimensionless contact load, dimensionless contact area in terms of E/Y ratio, Poisson's ratio and dimensionless interference then it is extended to analysis the contact behaviour of smooth to rough surface against a rigid flat surface. The results show that the contact load and contact area significantly deviate from other models and the surface parameters significantly influence the contact parameters.

Keywords: single asperity contact model, rough surface contact, effect of material property, plasticity index.

1. INTRODUCTION

In precision instruments, precision bearings and precision ball screws understanding the contact interaction of the components is important to enhance its performance. The contact surfaces of machine components are rough at microscopic level and understanding the deformation behavior of rough surface contacts is essential to minimize/maximize the tribological consequences of contacting components. In general, contact of rough surfaces is modeled and analyzed through statistical, deterministic and fractal approaches. In statistical approach, contacting spots (asperities) of rough surfaces are described in terms of shape, size, spacing and asperity height then theoretical expressions are developed to describe the deformation behavior of the asperities in contact and finally extending it to the whole surface contacts.

In this way, several statistical rough surface contact models have been proposed. A pioneer contact model is the Greenwood and Williamson [1] elastic contact model. Abbott and Firestone [2] developed a fully plastic contact known as surface microgeometry model. Chang et. al.,(CEB model) [3] bridged the fully elastic and fully plastic contact approaches by an elastic-plastic contact model on the basis of volume of conservation of plastically deformed asperities. This model adopted an abrupt transition from fully elastic to fully plastic state. The results showed that the mean separation is large and real area of contact is small in elastic-plastic contact than elastic contact for the same plasticity index and contact load. Zhao et. al.,(ZMC model) [4] devised an elastoplastic contact model, which interpolates the fully elastic to fully plastic states. This model used mathematical functions to smoothen the fully elastic, elastoplastic and fully plastic states. Smaller mean separation and larger real area of contact were predicted by the ZMC model than the GW model at any given plasticity index and contact load. ZMC model showed a complete elastic-plastic contact phenomena between rough surfaces for wide range of plasticity index and contact load compared to GW model and CEB model. The exact inception of elastoplastic to fully plastic was not governed. Kogut and Etsion (KE Model) [5,6] developed a FEM based elastic-plastic single asperity based rough surface contact model. This model developed generic empirical relationship for dimensionless mean contact pressure, dimensionless contact load and the dimensionless contact area with the dimensionless interference ratio. The results showed that the fully plastic deformation on the contact surface occurs at a constant dimensionless interference ratio of 110, at that stage the mean contact pressure ratio (P_{mean}/Y) reaches 2.8. They incorporate the single asperity finite element results to predict the contact parameters of rough surfaces, i.e., the mean separation, contact load, and the real area of contact in dimensionless forms. For calculating the contact parameters, they used the same hardness value throughout the statistical model, but they varied the standard deviation of surface heights and the plasticity index from 0.5 to 8, as in the CEB model. Their results were identical with CEB model till the plasticity index of 0.6 as pure elastic. The plasticity index of 1.4 marked as the transition of elastic to elastic-plastic and above the plasticity value of 8 entered to fully plastic. Jackson and Green [7,8] (JG model) extended the Kogut and Etsion work to account the geometry and material effects in the analysis. For calculating the critical interference, this model used von Mises yield criterion and material yield strength directly. This model formulated new empirical relationships to calculate contact load and contact area with respect to the deformation for elastic-perfectly plastic case based on the FEM results. The results showed that the mean contact pressure (p/Y) ratio does not reach 2.8 for most of the yield strength values. The end of the elastoplastic state is not identified. Further the empirical expressions are not updated for the general elastic–plastic cases. The developed empirical relations of dimensionless contact parameters are used to study the rough surface contact. In which, the plasticity index was varied from 0.5 to 100 by varying the material properties alone. They concluded that till the plasticity index of 10, the KE model and their model can be interchangeable but for high plasticity index values large

* Author to whom correspondence should be made, Email: shun@acer.iitm.ernet.in

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differences are observed. Brizmer et. al.,[9,10] conducted FEM based single asperity contact model under perfect slip and full stick conditions till the dimensionless interference ratio of 110. This model considered the materials with E/Y greater than 500 and E_t/E of 0.02 with Poisson's ratio of 0.25,0.35 and 0.45.The results showed the contact parameters are insensitive to contact conditions(perfect slip or full stick),independent of E/Y ,E_t/E, asperity radius but slightly depend on Poisson's ratio. This model didn't consider the high yield materials and the effect of tangent modulus. Shankar and Mayuram [11,12](SM model) extended the JG model to study the formation of elastic core, the transition of elastoplastic to fully plastic state and the effect of tangent modulus. The results showed that the elastic core formation, the maximum P_{mean}/Y ratio and the transition from elastoplastic to fully plastic state depend on E/Y and E_t/E ratios. The effect of Poisson's ratio is not considered. Sahoo et. al.,[13] extended SM model to account the effect of varying elastic modulus and asperity radius for wide range of dimensionless interference. The results showed that the contact parameters are independent of asperity radius but the maximum P_{mean}/Y ratio depends on E/Y ratio. Megalingam and Mayuram[14] (MM Model) extended the SM model to explore the exact transition of elastoplastic to fully plastic contact and start of fully plastic regime due to varying E/Y ratio and Poisson's ratio. They developed complex empirical relations for contact parameters in terms of dimensionless contact interference, E/Y ratio and Poisson's ratio but didn't extend to analysis the rough surface contacts.

The present work is an extension of MM model to explore the effect of rough surface contacts. Initially, for single asperity contact model, new empirical relations of dimensionless contact load and dimensionless contact are developed as function of dimensionless interference, E/Y ratio and Poisson's ratio based on the MM model results. Then, the contact parameters are extended for the statistical Gaussian rough surface contacts for varying plasticity index. The results are compared with the JG model and KE model and the salient features are discussed.

2. SINGLE ASPERITY CONTACT MODELS

One way of solving the microgeometric rough surface contacts is a statistical approach. In general the statistical approach represents a rough surface as an array of asperities with the shape and size while the asperity heights are assumed to vary randomly and follow a gaussian distribution(fig 1). This hemispherical model in contact with a rigid surface must be analysed. Then the analysed results of this single asperity model can be extended to a desired rough surfaces by gaussian distribution based on the applications.

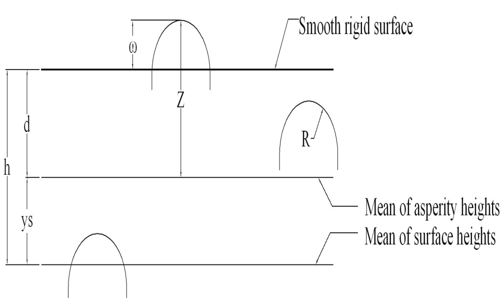

Fig. 1 Statistical distribution of asperities

2.1. Kogut and Etison Model (KE Model):

$$\omega_c = \left(\frac{\pi K H}{2E}\right)^2 R \qquad P_c = \frac{4}{3} E R^{\frac{1}{2}} \omega_c^{\frac{3}{2}}$$

$$K = 0.454 + 0.41\upsilon \qquad A_c = \pi R \omega_c$$

$$\left(\frac{P}{P_c}\right)_1 = 1.03\left(\frac{\omega}{\omega_c}\right)^{1.425}$$

$$\left(\frac{A}{A_c}\right)_1 = 0.93\left(\frac{\omega}{\omega_c}\right)^{1.136} \quad 1 \le \frac{\omega}{\omega_c} \le 6$$

$$\left(\frac{P}{P_c}\right)_2 = 1.40\left(\frac{\omega}{\omega_c}\right)^{1.263}$$

$$\left(\frac{A}{A_c}\right)_1 = 0.94\left(\frac{\omega}{\omega_c}\right)^{1.146} \quad 6 \le \frac{\omega}{\omega_c} \le 110$$

2.2. Jackson and Green Model (JG Model):

$$\omega_c = \left(\frac{\pi C S_y}{2E*}\right)^2 R \text{, where } C = 1.295\exp(0.736\nu)$$

$$P_c = \frac{4}{3}\left(\frac{R}{E^*}\right)^2 \left(\frac{C}{2}\pi S_y\right)^3 \qquad A_c = \pi^3 \left(\frac{C S_y R}{2E^*}\right)^2$$

Where

$$B = 0.14\exp\left(23.e_y\right) \qquad e_y = \frac{S_y}{E'} \text{ for } \omega \ge 1.9\omega_c$$

$$\frac{H_G}{S_y} = 2.84\left[1 - \exp\left(-0.82\left(\sqrt{\frac{\omega}{R}}\left(\frac{\omega}{1.9\omega_c}\right)^{B/2}\right)^{-0.7}\right)\right]$$

2.3. Present model:

$$\omega_c = C_v\left(\frac{\pi(1-\upsilon^2)}{2}\left(\frac{Y}{E}\right)\right)^2 R$$

$$P_c = \left(\frac{\pi^3 c_v^{\,3} Y}{6}\right)\left(\frac{R(1-\upsilon^2)Y}{E}\right)^2$$

$$C_v = 1.234 + 1.256\upsilon$$

$$A_c = \pi R \omega_c$$

2.4. Dimensionless contact load

$$79 \le \frac{E}{Y} \le 356 \qquad 1 \le \frac{\omega}{\omega_c} \le 80 \qquad (0.25 \le \upsilon \le 0.45)$$

$$\ln\left(\frac{p}{p_c}\right) = -228.33356 + \left(-0.055219082 * \left(\ln\frac{\omega}{\omega_c}\right)^2\right) + \left(1.60915 * \ln\left(\frac{\omega}{\omega_c}\right)\right) + \left(-115.11537 * \left(\frac{E*\upsilon}{Y}\right) * \ln\left(\frac{E*\upsilon}{Y}\right)\right) + \left(149.09703 * \left(\frac{E*\upsilon}{Y}\right)^{1.5}\right) + \left(-28.332089 * \left(\frac{E*\upsilon}{Y}\right)^2\right)$$
$$+ \left(4.7886349 * \left(\frac{E*\upsilon}{Y}\right)^2 * \ln\left(\frac{E*\upsilon}{Y}\right)\right) + \left(-0.35523233 * \left(\frac{E*\upsilon}{Y}\right)^{2.5}\right) + \left(0.0024629848 * \left(\frac{E*\upsilon}{Y}\right)^3\right)$$

$$80 \le \frac{\omega}{\omega_c} \le 450$$

$$\left(\frac{p}{p_c}\right) = -6036.5957 + \left(3670.4292 * \ln\left(\frac{\omega}{\omega_c}\right)\right) + \left(32.202452\left(\frac{E*\upsilon}{Y}\right)\right) + \left(-674.73604 * \left(\ln\left(\frac{\omega}{\omega_c}\right)\right)^2\right) + \left(0.74688724 * \left(\frac{E*\upsilon}{Y}\right)^2\right) + \left(-33.327913 * \left(\frac{E*\upsilon}{Y}\right) * \ln\left(\frac{\omega}{\omega_c}\right)\right)$$
$$+ \left(40.252492 * \left(\ln\left(\frac{\omega}{\omega_c}\right)\right)^3\right) + \left(-0.0004781715\left(\frac{E*\upsilon}{Y}\right)^3\right) + \left(-0.14241105 * \ln\left(\frac{\omega}{\omega_c}\right) * \left(\frac{E*\upsilon}{Y}\right)^2\right) + \left(5.48847 * \left(\frac{E*\upsilon}{Y}\right) * \left(\ln\left(\frac{\omega}{\omega_c}\right)\right)^2\right)$$

$$356 \le \frac{E}{Y} \le 958 \qquad 1 \le \frac{\omega}{\omega_c} \le 80$$

$$\ln\left(\frac{p}{p_c}\right) = 69502.139 + \left(-0.038594463 * \ln\left(\frac{\omega}{\omega_c}\right)^2\right) + \left(1.5394945 * \ln\left(\frac{\omega}{\omega_c}\right)\right) + \left(1340.2188 * \ln\left(\frac{E*\upsilon}{Y}\right)^2\right) + \left(-169.1952 * \left(\frac{E*\upsilon}{Y}\right)^{0.5}\right) + \left(-18801.906 * \ln\left(\frac{E*\upsilon}{Y}\right)\right) + \left(\frac{84031.652}{\ln\left(\frac{E*\upsilon}{Y}\right)}\right)$$
$$+ \left(\frac{-373724.38}{\left(\frac{E*\upsilon}{Y}\right)^{0.5}}\right) + \left(205813.14\left(\frac{\ln\left(\frac{E*\upsilon}{Y}\right)}{\left(\frac{E*\upsilon}{Y}\right)}\right)\right)$$

$$\ln\left(\frac{p}{p_c}\right) = -117071.67 + \left(\frac{-109.02659}{\ln\left(\frac{\omega}{\omega_c}\right)}\right) + \left(\frac{77.865891}{\left(\frac{\omega}{\omega_c}\right)^{0.5}}\right) + \left(1.15892396 * 10^{-6}\left(\frac{E*\upsilon}{Y}\right)^3\right) + \left(\frac{1062298}{\ln\left(\frac{E*\upsilon}{Y}\right)}\right) + \left(-29507528\left(\frac{\ln\left(\frac{E*\upsilon}{Y}\right)}{\left(\frac{E*\upsilon}{Y}\right)}\right)\right) + \left(\left(\frac{1.9697362 * 10^8}{\left(\frac{E*\upsilon}{Y}\right)}\right)\right)$$
$$+ \left(\frac{-1.18632585 * 10^9}{\left(\frac{E*\upsilon}{Y}\right)^{1.5}}\right) + \left(1.0029658 * 10^9\left(\frac{\ln\left(\frac{E*\upsilon}{Y}\right)}{\left(\frac{E*\upsilon}{Y}\right)^2}\right)\right)$$

2.5. Dimensionless Contact Area

$$79 \le \frac{E}{Y} \le 958 \qquad 1 \le \frac{\omega}{\omega_c} \le 80 \quad (0.25 \le \upsilon \le 0.45)$$

$$\left(\frac{A}{A_c}\right)^{-1} = -2.7700138 + \left(-0.98294541\left(\frac{\ln\left(\frac{\omega}{\omega_c}\right)}{\left(\frac{\omega}{\omega_c}\right)}\right)\right) + \left(\frac{0.9575097}{\left(\frac{\omega}{\omega_c}\right)}\right) + \left(-0.025575525 * \left(\frac{E*\upsilon}{Y}\right)^{0.5} * \ln\left(\frac{E*\upsilon}{Y}\right)\right) + \left(-0.14589492 * \left(\ln\left(\frac{E*\upsilon}{Y}\right)\right)^2\right) + \left(0.30367008 * \left(\frac{E*\upsilon}{Y}\right)^{0.5}\right)$$
$$+ \left(0.82909436 * \ln\left(\frac{E*\upsilon}{Y}\right)\right) + \left(\frac{-1.0616582}{\ln\left(\frac{E*\upsilon}{Y}\right)}\right) + \left(\frac{4.180017}{\left(\frac{E*\upsilon}{Y}\right)^{0.5}}\right)$$

$$80 \le \frac{\omega}{\omega_c} \le 450$$

$$\left(\frac{A}{A_c}\right)^{-1} = -0.066643587 + \left(\frac{0.40466773}{\left(\frac{\omega}{\omega_c}\right)}\right) + \left(\frac{1.3112678}{\left(\frac{\omega}{\omega_c}\right)^{1.5}}\right) + \left(\frac{8.0533218}{\left(\frac{E^*\upsilon}{Y}\right)^{0.5}}\right) + \left(-99.171604\left(\frac{\ln\left(\frac{E^*\upsilon}{Y}\right)}{\left(\frac{E^*\upsilon}{Y}\right)}\right)\right) + \left(\frac{570.45608}{\left(\frac{E^*\upsilon}{Y}\right)}\right) + \left(\frac{-2799.495}{\left(\frac{E^*\upsilon}{Y}\right)^{1.5}}\right) + \left(1792.1313\left(\frac{\ln\left(\frac{E^*\upsilon}{Y}\right)}{\left(\frac{E^*\upsilon}{Y}\right)^{2}}\right)\right) + \left(\frac{979.26285}{\left(\frac{E^*\upsilon}{Y}\right)^{2}}\right)$$

2.6. Rough Surfaces Contact:

Based on single asperity contact model with Greenwood and Williamson model assumptions, contact parameters like total contact area, total contact load with mean separation are to be calculated. The empirical relations used to model the rough surface contact is discussed below

Some base parameters that should be considered between contacting rough surfaces are explained. Mainly two reference planes can be defined. (i.e., mean of asperity heights and mean of the surface heights). Let z and d are the asperity heights and separation of the surfaces with R is the radius of the asperity. h is the separation of the surfaces from the reference planes.

All the models utilized Gaussian distribution for the asperity height distribution and that is given as

$$\phi^*(z^*) = \frac{1}{\sqrt{2\pi}}\frac{\sigma}{\sigma_s}\exp\left[-0.5\left(\frac{\sigma}{\sigma_s}\right)^2 (z^*)^2\right]$$

The standard deviation σ_s and σ correspond to the asperity and surface heights, respectively and are related as :

$$\sigma^2 = \sigma_s^2 + \frac{3.717*10^{-4}}{\eta^2 R^2}$$

All the dimensions are normalized by σ and the dimensionless values are denoted by *. $\omega_c^* = \frac{\omega_c}{\sigma}$

Critical interference can be normalized with σ to give:

y_s^* is the difference between h* and d* and is calculated by

$$y_s^* = h^* - d^* = \frac{1.5}{\sqrt{108\pi\beta}}$$

The number of asperities on the contacting surface can be found by multiplying the nominal surface by the area density of the asperities:

$$N = \eta A_n$$

Then, the total number of asperities in contact is defined as:

$$N_c = \eta A_n \int_d^\infty \phi(z)dz$$

The individual asperity contact area, A, and force, P, are functions of each asperity's interference, ω. Thus, the contribution of all asperities of a height z to the total contact area and total contact force can be calculated as:

$$A(z) = \eta A_n A(z-d)\phi(z)$$

$$P(z) = \eta A_n P(z-d)\phi(z)$$

Then, the total area of contact and total contact force between the surfaces is found by simply integrating the above equation over the entire range of asperity contact:

$$A(d) = \eta A_n \int_d^\infty A(z-d)\phi(z)dz$$

$$P(d) = \eta A_n \int_d^\infty P(z-d)\phi(z)dz$$

Greenwood and Williamson defines plasticity index to relates the critical interference and the roughness of the surface to the plastic deformation of the surface and the relation is

$$\psi = \sqrt{\frac{\sigma_s}{\omega_c}}$$

Here in this work by holding material properties as constant and surface roughness is varied.

3. RESULTS AND DISCUSSION

The surface roughness is varies from $9.0\text{x}10^{-9}$m to $0.17\text{x}10^{-9}$m [3] by keeping all other parameters as constant. The considered material properties are E=210GPa, Poisson's ratio 0.32,yield strength 579MPa form the JG model[7].

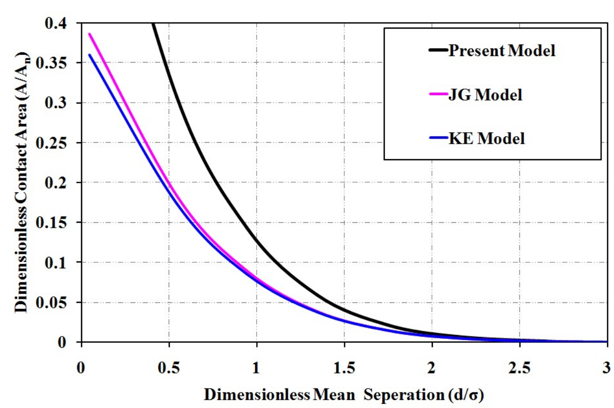

Fig. 2 Dimensionless contact area vs Mean separation for plasticity index 10.0

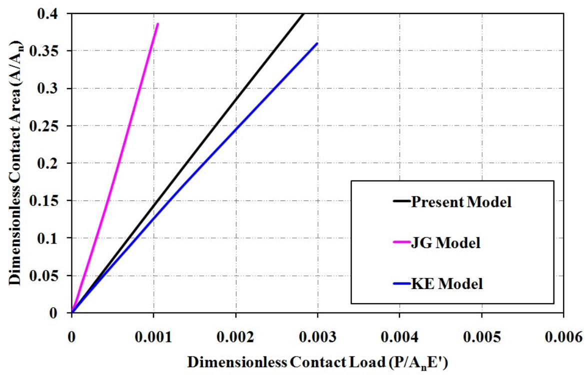

Fig. 3 Dimensionless contact area vs Dimensionless contact load for plasticity index 10.0

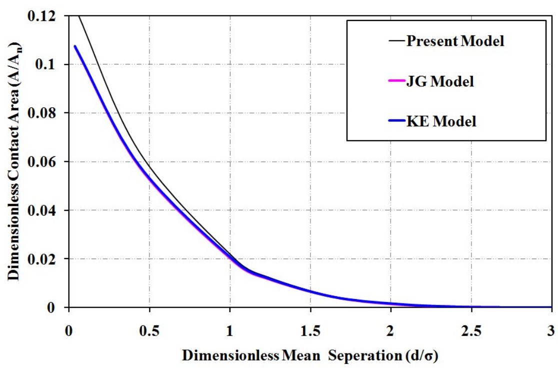

Fig.4 Dimensionless contact area vs Mean separation for plasticity index 6.0

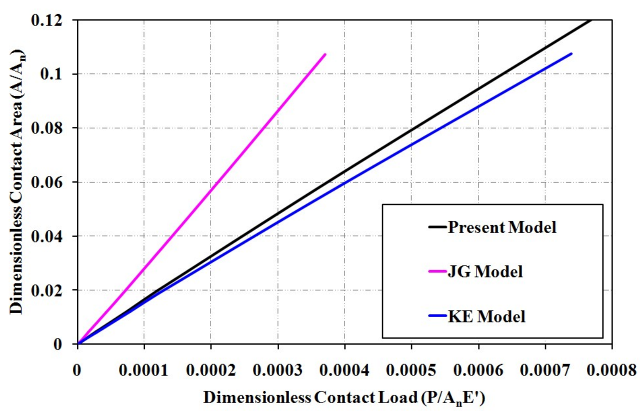

Fig. 5 Dimensionless contact area vs Dimensionless contact load for plasticity index 6.0

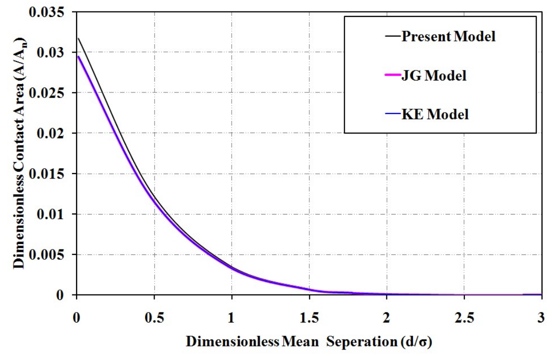

Fig. 6 Dimensionless contact area vs Mean separation for plasticity index 1.22

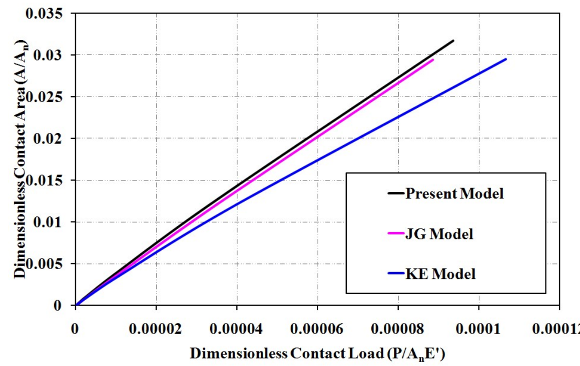

Fig. 7 Dimensionless contact area vs Dimensionless contact load for plasticity index 1.22

The figures 2, 4 and 6 show the variation of contact area due to the decreasing plasticity index. For high values of plasticity index, the present model contact area is large compared to KE and JG models because the present model area calculation include the effect of E/Y ratio and Poisson's ratio based MM model. The figures 3, 5 and 7 show the variation of dimensionless contact area with the dimensionless contact load. For high plasticity index values, the JG model overestimates whereas the KE model underestimates due to its limits of W/Wc as 110. As the plasticity index decreases the present model and JG model follows as close relation whereas the KE model underestimates.

4. CONCLUSION

* The present model developed empirical relations for the dimensionless contact load and dimensionless contact area as function of E/Y ratio and Poisson's ratio based MM model.

*The present model single asperity contact parameters are extended to study the rough surface contact of Gaussian surfaces with varying plasticity index of 10 to1.22

*The present model rough contact parameters calculations are close with JG model whereas large deviation was marked with KE model due to its limitation.

*The present model contact parameters can be extended to study the non Gaussian rough surface contacts appropriately.

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