Estimation of Modal Parameters of a LegacyVertical Milling Machine based on Random Cutting Excitation through Operational Modal Analysis

D. Poddar and M.S. Shunmugam*
Manufacturing Engineering Section, Department of Mechanical Engineering
Indian Institute of Technology, Chennai - 600 036, INDIA

Abstract

In high performance machining, the dynamic characteristics of machine tool structure greatly limits the quality of machined parts. Chatter free regions of a stability lobe diagram are predicted based on the dynamic parameters. However, these parameters are known to be notoriously variable between static test conditions and metal cutting operational conditions.
Operational Modal Analysis, a well-established tool in Civil Engineering, can be exploited for modal parameter estimation of machine tools from cutting force excitation. Thisrequires white noise input excitation to perform both time/frequency domain decompositions. This present work proposes a novel random workpiece design that can generate the required flat force spectrum on a legacy machine which allows limited spindle speeds and feeds. Effect of various cutting parameters on the force spectrum is analysed and simulated. Cutting experiment on a vertical milling machine was performed to realize the proposal. A complete modal decomposition methodology is described using Autoregressive, Autoregressive Moving Average and Least Squares Complex Exponentialnumerical approaches. Analytical dynamic stability diagrams are generated based on the numerical analysis.

Keywords: Operational Modal Analysis, Dynamic Stability, Vertical Milling Machine

1. INTRODUCTION

High quality machining requires stability of the intertwined machine-tool and machining dynamic system so as to avoid chatter which produces poor surface finish and noise. Choosing of stable cutting conditions is done with the aid of stability lobe diagrams. These diagrams are most often formed with the dynamic parameters (dominant natural frequencies and corresponding damping ratios) estimated from impact/shaker testdatausing Experimental Modal Analysis. However it is well established that dynamic parameters change during machining compared to the static case, due to changes in spindle speed, boundary conditions at tool-workpiece interface and hydrodynamic conditions of lubricants with feed [1]. Performing tap test on a rotating spindle would indeed be hazardous.Moreover, there is difficulty in repeatability of impact testing and input excitation energy is highly operator dependant [2].Therefore, there are significant difficulties along the EMA route.

The cutting force itself provides an intuitively appealing,alternative input excitation source.However the strong tooth passing frequency content would overshadow other frequencies. Thus various researchers have proposed varied and novel schemes to generate broadband white excitation. Opitz and Weck [3], Minis [4]have proposed random workpieces for turning. Bin Li et. al. [2] have proposed random spindle speed variation for face milling. Random feeds have also been considered in [5].

*Author to whom correspondence should be made,
Email: shun@iitm.ac.in

Many of such methods estimate FRFs based on measurement of both input excitation and output acceleration. It requires relatively costly dynamometers which permit limited workpiece sizes and its use can be challenging with complex fixturing.
Operational Modal Analysis (OMA), a powerful toolkit from civil engineering, has been adopted into machine tool research to obviate the above problems. OMA is an output only modal analysis which operates under the assumption of ambient or white noise input. Various time domain methods like ARMA (Autoregressive Moving Average), LSCE (Least Squares Complex Exponential), ITD (Ibrahim Time Domain) and frequency domain methods like PolyMax and FDD (Frequency Domain Decomposition) have been successfully applied to machine tool vibration analysis.
In the present study a novel random workpiece design is proposed along with a complete experimental and numerical methodology to extract modal parameters with ease in the shop floor on a variety of machine tools without costly dynamometer.

Nomenclature:

R_{ijk}	Autocorrelation function
X(n)	Cross correlation function of measured accelerations
t	Time in sec
T	Lags in sec
x	Acceleration signal
ζ	Coefficient of damping
ω^d_r	Damped natural frequency
α, β	AR/LSCE parameters
i, j, k	Locations

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2. OMA

When a structure is excited by a random zero mean white noise input excitation at location k, the cross correlation of the output signals measured at locations i and j (x_{ik} and x_{jk}) are given by equation (1) [6].

$$R_{ijk}\left(T\right) = E\left[x_{ik}\left(t+T\right)x_{jk}\left(t\right)\right] \quad (1)$$

On expanding the expectation operator and substituting the output response as convolution of impulse response function and input force and using the fact that the autocorrelation of white noise is the delta function, leads to equation (2) after simplification.

$$R_{ijk}(T) = \sum_{r=1}^{N} G_{ijk}^{r} e^{-\zeta_r \omega_r T} \cos(\omega_r^d T) + H_{ijk}^{r} e^{-\zeta_r \omega_r T} \sin(\omega_r^d T) \quad (2)$$

Thus the cross correlation function can be represented as a summation of N decaying sinusoids with characteristic frequency $\omega_r$and damping ratio ζ_r. Therefore this is similar to Impulse Response Function and can be considered as free decays as far as time domain identifications are concerned.

3. WORKPIECE DESIGN

3.1. Cutting process-Machine tool loop

During machining (face milling here), the cutting process and machine tool structure interact dynamically to form a feedback loop. This presents difficulties in system identification. However, in the special case of *interrupted cutting*, the motion of the cutter is a free vibration within the interval of consecutive tooth engagements. This combined with the assumption of linear, controllable structure, decouples the machine tool structure dynamics from the cutting process and allows us to proceed with identification despite the presence of the feedback loop [4].

3.2. 'Random'workpiece generation

As was seen in the previous section, one of the central requirements of OMA was the input excitation to be representable as white noise. This can be achieved in many ways in a vertical milling machine including modifying the workpiece or the cutter, or continuously modifying the spindle speed or feed. In the present study, the object of identification was a legacy vertical milling machine which provides only a few speeds and feeds through gear trains and manual shifts. No facility for numeric control exists. Also, modification of tool can lead to unstable cutting with the risk of tool breakage. Thus modification of workpiece was selected as the preferred route.
A couple of random workpiece were designed (Fig. 1.) which

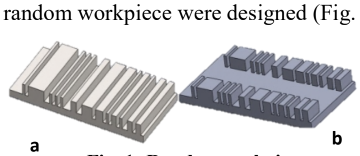

Fig. 1: Randomworkpiece

has grooves (0) and lands (1) distributed along its length based on a binary pseudo-random sequence. This is achieved by using feedback shift registers (representing a primitive polynomial of order m=6 as in equation (3)) to create a sequence of length 2^m-1 [7]. Such sequences have auto-correlation given by equation (4) which is the best possible value forpseudo-sequences of such lengths.

$$h(x) = x^6 + x + 1 \quad (3) \text{ and } \rho\left(i\right) = -\frac{1}{2^m - 1} \quad (4)$$

3.3. Effect of cutting parameters and simulation

For ease of manufacture, the narrowest groove of the workpiece was made 6 mm which can be cut with a 6 mm end mill. The total cutting length was decided so that the a minimum time series length isabout 20 times the memory of the lowest frequency of interest[6] as shown in equation (5). Thus the effect of feed is incorporated in workpiece design.

$$T_{cutting} > \frac{20}{2\zeta_{min}} = \frac{10}{\zeta_{min}} \quad - \quad (5)$$

The path followed by each tooth will be a trochoid. Superposition of such trochoids for multiple teeth will provide the cutting path. The bite of each tooth into the workpiece produces a practical impulse of a finite magnitude and small duration τ. The cutting process thus leads to a chain of such impulses (Fig. 2). The Power Spectral Density of such a force becomes the summation of the squaredsinc function [1]. This shows a lobed nature. But the length of the lobes could be controlled by controlling the median of duration τ which is a function of workpiece design and spindle speed. If the lobes are sufficiently broad, the spectrum becomes white within the lobed band (Fig. 3).

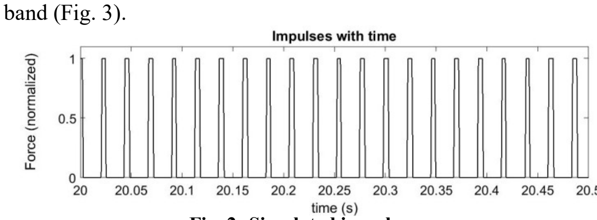

Fig. 2: Simulated impulses

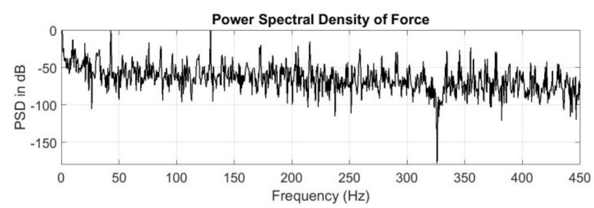

Fig. 3: Simulated Force PSD

4. EXPERIMENTAL VERIFICATION

4.1. Setup

Experimentation was performed on a Fritz Werner Vertical Milling Machine FV3D with face milling of a random MS workpiece with cutter of diameter 100 mm having seven 1203 carbide Face Mill inserts (Fig. 4). 2 three axes accelerometerDYTRAN 3053B with DAQ: LDS DactronPhoton+(Sampling Freq=1024 Hz) were used for measuring acceleration signals at the spindle housing (loc. i=a_{xi}/a_{yi}) and dynamometer (loc. j=a_{xj}/a_{yj}). Kistler Dynamometer 9257B was used for force measurement with dedicated Kistler DAQ (F_s=1024 Hz). This created a problem of aligning the acceleration and force signals to a common time before analysis could proceed.

Following [8], this alignment (Fig. 5) was achieved by finding the cross correlation of the force and corresponding acceleration in a non-feed direction. The laggard signal was forwarded by an amount equal to the time lag corresponding to the maximum of the cross correlation.

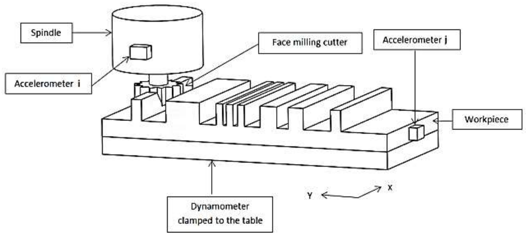

Fig. 4: Experimental setup

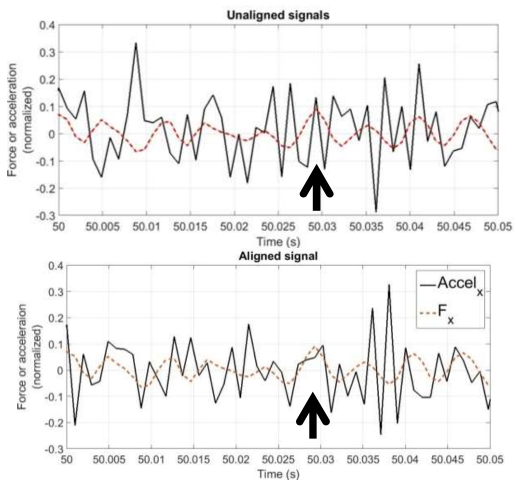

Fig. 5: Alignment of signals

The cutting conditions are summarized in Table 1. Test 1 was used for parameter extraction.

Table 1: Cutting Conditions

#	Feed(mm/min)	Axial Depth(mm)	Spindle Speeds (rpm)
1	100	0.4	355
2	100	0.5	355
3	100	0.6	355
4	340	0.5	355
5	500	0.5	355
6	100	0.5	281
7	100	0.5	425

4.2. Auto-Regressive model

In an AR (Auto-Regressive) process the input and output relationship is described as equation (6) where X(n) is the observed real output sequence, e(n) is the zero mean random variable, a(k) are the AR parameters and p is the order of the model.X(n) here is the cross correlations between acceleration signals from i and j in both x and yi.e.R_{ijx}= cross-corr(a_{ix},a_{jx}) and R_{ijy}= cross-corr(a_{iy},a_{jy}).

$$X(n) = -\sum_{k=1}^{p} a(k)X(n-k) + e(n) \tag{6}$$

The FRF in this case is given by equation (7) [9].

$$H(e^{i2\pi f}) = \frac{1}{1+\sum_{k=1}^{p} a(k)e^{-i2\pi fk/F_s}} \tag{7}$$

The characteristic equation of the system is given as equation (8) whose roots are z_r and the modal parameters (f_r and ζ_r) can be extracted by equation (9) and (10). a(k) are found by solving the Yule–Walker equation by Durbin-Levinson algorithm

$$1+\sum_{k=1}^{p} a(k)z^{-k} = 0 \tag{8}$$

$$f_r = \frac{F_s}{2\pi}\sqrt{\ln(z_r)\ln(z_r^*)} \tag{9}$$

$$\zeta_r = -\frac{F_s}{4\pi f_r}\sqrt{\ln(z_r z_r^*)} \tag{10}$$

An AR process order is identified with a cut off in the Partial Autocorrelation function as can be observed forR_{ijx} in Fig. 6.

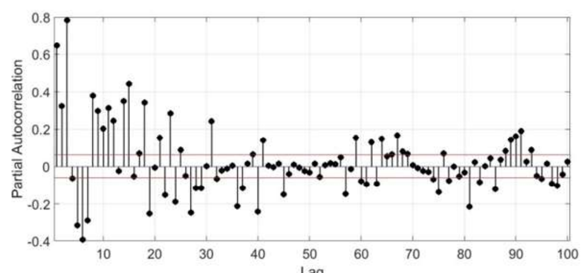

Fig. 6: PACF of accelerations in X

The identified resonant frequencies have been plotted with model order and resonances of both x (º) and y (+) axis are shown simultaneously in Fig. 7.

The tooth passing and noise modes are eliminated from the modelbased onZaghbani and Songmene criteria [1]:

- Complete superposition of x and y frequencies would indicate tooth passing frequencies.
- Very low damping of <0.05% indicates spurious mode.

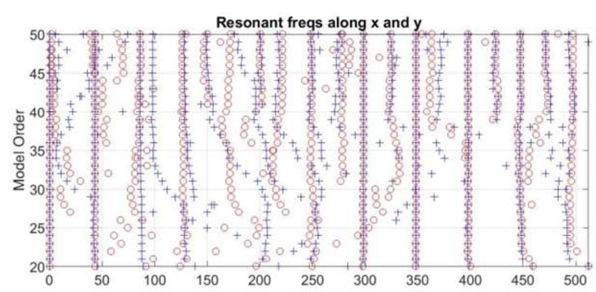

Fig. 7: Stabilization diagrams of AR

The results are summarized in Table 2.

4.3. Least Squares Complex Exponential model

In LSCE method, the $z_r$roots are found by solving the Prony's equation (equation (11)) and extracting modal data by equations (9) and (10) [9].

$$\beta_0 + \beta_1 z_r + \beta_2 z_r^2 + \cdots + \beta_{2N-1} z_r^{2N-1} + \beta_{2N} z_r^{2N} = 0 \quad (11)$$

Here 2N gives the modal order. $\beta_{2N}$is set as 1 and from a series of simultaneous equations involving the cross correlation of accelerations of 2 stations i and j(R_{ijx} or R_{ijy})we get the β coefficients by equation (12). The number of equations can be increased and the system solved by Least Square method.

$$\begin{pmatrix} R_{ij}(0) & \ldots & R_{ij}(2N-1) \\ \vdots & \ddots & \vdots \\ R_{ij}(2N-1) & \cdots & R_{ij}(4N-2) \end{pmatrix} \left(\beta_0 \beta_1 \beta_2 \ldots\right)^T = \left(R_{ij}(2N) R_{ij}(2N+1) \ldots\right)^T \quad (12)$$

Similar to the AR model, a stabilization diagram of resonant frequenciescan be constructed.

4.4. ARMA model

In an ARMA (p,q) model as in equation (13), the characteristic equation has same form as equation (8) and modal extraction is proceeded in a similar fashion to obtain f_r and ζ_r [9].

$$X(n) = -\sum_{k=1}^{p} a(k) X(n-k) + e(n) + \sum_{k=1}^{q} b(k) e(n-k) \quad (13)$$

Setting q=2 is known to improve modelling quality. One major difference with the AR(p) model is that the time domain data is directly fitted to the model without going through the Cross-correlation route. Here X(n) is the relative acceleration data found by the difference of the acceleration signals in both x and y directions i.e. a_{ix}- a_{jx} and a_{iy}- a_{jy}[9].

Table 2: Modal Parameters along feed direction

Freq. (AR)	Damping %	Freq. (ARMA)	Damping %	Freq. (LSCE)	Damping %
105.2	13.8	106.5	8.5	100.3	5.9
157.6	5.4	151.3	6.9		
196.7	1.7	194.8	4.6	199.7	1.6
228.6	1.8	227.6	3.3	233.6	1.2
268.3	1.4	269.3	3.0	272.7	2.9
282.3	0.9	283.2	2.4	287.9	2.8
358.4	0.9	357.8	0.6	359.0	2.2

4.5. Stability lobe diagram

The characteristic root having the lowest absolute value of the real part has significant contribution to the dynamics of the system [9]. In fact researchers have shown that a plot of this quantity versus spindle speeds or feed produces a numerical stability lobe diagram whose shape is strikingly similar to the experimentally generated stability lobe diagram. Such a diagram is shown in Fig. 8 where stability (Y) is plotted against feed per tooth. The lobed nature of the curve is observed and is similar to those in literature [10].

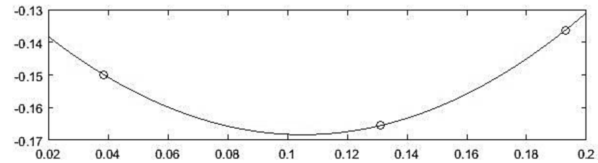

Fig. 8: Stability with feed/toothvariation

5. CONCLUSION

A complete methodology for easy modal analysis of legacy machines without expensive equipment and easily performable on the shop floor was sketched. A novel workpiece design was discussed which was shown by simulation and experiment to be capable of generating broadband white noise excitation as required by OMA. AR, LSCE were used in extracting the modal parameters. Stability variation with feed was also demonstrated. Further investigation of the effect of rpm is contemplated as well as using a higher density of accelerometers for a more comprehensive picture of the total machine tool. Also reasons for spread in resonance in different methods will be studied.

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